Fuzzy C-Means Algorithm Based on Gaussian Function for Magnetic Resonance Images (MRIs) Segmentation

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Abstract

In this paper, we propose fuzzy c-means (FCM) method based on Gaussian function for improving magnetic resonance imaging (MRI) segmentation. The proposed algorithm is formulated by modifying the objective function of the standard FCM algorithm to allow the labeling of a pixel to be influenced by other pixels to suppress the noise effect during segmentation processes. The proposed algorithm is feed by the initial centers for the objective function as a prior knowledge to avoid the coincident clusters. Then, the process of finding the best clusters are continue to update the centers and the membership and only stop when the factor between two successive centers is smallest than a prescribed value. The proposed algorithm is applied to magnetic resonance image (MRI) datasets. Compared with the existing approaches, the proposed method can achieve the best accurate results.

Keywords: Fuzzy clustering, modified fuzzy c-means, medical image segmentation.

1 Introduction

Because of the advantages of magnetic resonance imaging (MRI) over other diagnostic imaging [1-2], the majority of researches in medical image segmentation pertains to its use for MR images. Fuzzy segmentation methods have considerable benefits, because they could retain much more information from the original image than hard segmentation methods [3]. In particular, the fuzzy c-means (FCM) algorithm [1] assigns pixels to fuzzy clusters without labels. Since the conventional FCM algorithms classify pixels in the feature space without considering their spatial distribution in the image, it is highly sensitive to noise and other imaging artifacts. Many extensions of the FCM algorithm have been proposed to overcome above

mentioned problem and reduce errors in the segmentation process [4-11]. Among them, algorithms-based on modified FCM objective function is widely used in medical image clustering to suppress the noise effect during the segmentation processes. Modified FCM objective function is to add penalty term into the objective function to constrain the membership values. Based on the traditional FCM objective function, most improved approaches embodied regularization terms to show the increased robustness of the classification of the noisy images. Pham and Prince [12] modified the FCM objective function by introducing a spatial penalty for enabling the iterative algorithm to estimate spatially smooth membership functions. Ahmed et al. [5] introduced a neighborhood averaging additive term into the objective function of FCM. They named the algorithm bias corrected FCM (BCFCM). Liew and Yan [13] introduced a spatial constraint to a fuzzy cluster method where the inhomogeneity field was modeled by a b-spline surface. The spatial voxel connectivity was implemented by a dissimilarity index, which enforced the connectivity constraint only in the homogeneous areas. This way preserves significantly the tissue boundaries. Szilágyi et al. [14] modified the FGFCM (MFGFCM) to improve the precision of segmentation. They proposed EnFCM algorithm to accelerate the image segmentation process. EnFCM is based on a simple fact about images, which is usually overlooked in many FCM-type algorithms. Cai et al. [8] introduced a new local similarity measure by combining spatial and gray level distances. They used their method as an alternative pre-filtering to EnFCM. They named this approach fast generalized FCM (FGFCM). This method is able to extract local information that causes less blur than averaging filter. However, it still has an experimentally adjusted parameter and the precision of the segmentation is not good enough. Kang et al. [15] improved FCM with adaptive weighted averaging filter (FCM AWA). Kang et al. [16] proposed a spatial

homogeneity-based FCM (SHFCM). Wang et al. [17] incorporated both the local spatial context and the nonlocal information into the standard FCM cluster algorithm. They used a novel dissimilarity measure in place of the usual distance metric. These approaches could overcome the noise impact, but the intensity homogeneity cannot be handled at the same time. FCM-based algorithms are known to be vulnerable to outliers and noise. To address this problem, possibilistic clustering which is pioneered by the possibilistic c-means (PFCM) algorithm [18] is developed. It has shown more robust to outliers than FCM. However, the robustness of PFCM comes at the expense of the stability of the algorithm [19]. The PCM-based algorithms suffer from the coincident cluster problem, which makes them too sensitive to initialization [19]. Many efforts have been presented to improve the stability of possibilistic clustering [20, 21, 22]. However, PFCM estimates the centroids robustly in the case of outliers.

Although suppressing the impact of noise and intensity inhomogeneity to some extent, these algorithms still produces misclassified small regions [23-26]. They still depend on a fixed spatial parameter which needs to be adjusted. Furthermore, the cost of estimating the neighbors for each point in an image is still high. Therefore, these drawbacks will reduce the clustering performance in real applications.

This paper addresses these problems for overcoming the shortcomings of existing fuzzy methods. In order to reduce the noise effect during segmentation, a new fuzzy c-means algorithm based on Gaussian function is presented that could improve the medical image segmentation. The proposed algorithm is realized by modifying the objective function of the conventional FCM algorithm with a Gaussian function and to allow the labeling of a pixel to be influenced by its neighbors in the image. This function is feed by initial centers for the generation of fuzzy terms.

The efficiency of the proposed algorithm is demonstrated by extensive segmentation experiments using real MR images and by comparison with other state of the art algorithms.

The rest of this paper is organized as follows. In section 2, the proposed algorithm is presented. Experimental comparisons are given in section 3. Finally, Section 4 gives our conclusions..

2 The Proposed Algorithm

The choice of an appropriate objective function is a key to the success of cluster analysis and to obtain better quality clustering results; hence, clustering optimization is based on the objective function [22]. To identify a suitable objective function, one may start from the following set of requirements: the distance between the data points assigned to a cluster should be minimized and the distance between clusters should to be maximized [20]. To obtain an appropriate objective function, we take into consideration the following:

- The distance between clusters and the data points allocated to them must be reduced.
- Coincident clusters may occur and must to be controlled.
- Selecting the initialization sensitive parameters for decreasing noises affect.

To overcome the limitation of the fuzzy methods, we present a novel fuzzy c-means algorithm based on gaussian function. As for the common value used for this parameter by every data for iterations, we propose a new weight function which is based on Gaussian membership of a point p achieving every point of the data set has a weight in relation to every cluster. The usage of weights produces good classification particularly in the case of noisy data.

The proposed algorithm starts by partitioning the image into *C* regions of intensity by known the minimum and maximum values of intensity using well-known histogram algorithm [29]. The median point of each region R_k (including points $x_i, i = 1, 2, ... N_k$, N_k is the number of points of R_k) is selected to be as initial centers of the region, and then both region and centers are fed to the method. While the constraints term $\sum_{k=1}^{C} \sum_{i=1}^{N} \alpha u_{ki}^m e^{-\eta (x_i - c_k)^2}$

is only considered in the objective function if a point x_i belongs to a region R_k , k=1,2,..., C with initial center c_k which takes \hat{c}_k for the first iterative i.e. we only estimate this term if $x_i \in R_k$. The cost of computations can be reduced using a region and not neighbors for all points.

The objective function of the proposed modified fuzzy cmeans is modified to:

$$J_{m} = \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m} \|x_{i} - c_{k}\|^{2} + \sum_{k=1}^{C} \sum_{i=1}^{N} \alpha u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}}$$
(1)

where

 $u = (u_{ki})_{C \times N}$, $C = (c_1, c_2, ..., c_k)$, $\eta = 1/2\sigma^2$, σ is the standard deviation and $\alpha = 1/\sigma\sqrt{2\pi}$ is a constant.

However, the crucial parameter is m which represents the power of the memberships. More datasets are experimented in [27-28], they proved that there is a relation between data shape and m. For instance, the triangular shape will fit better if m=3 is used, more discussion can be shown in [28].Therefore we take into account the data shape in the objective function and to be

general for all tested data sets. This penalty term also contains prepressing centers information, which acts as a regularizer and biases the solution toward piecewise-homogeneous labeling. Such regularization is helpful in segmenting images corrupted by noise. The objective function J_m under the constraint of u_{ki} and C_k can be solved by using the following theorem [5]:

Theorem: Let $X = \{x_i, i = 1, 2, ..., N \mid x_i \in R^d\}$ denotes an image with N pixels to be partitioned into C classes (clusters), where X_i represents feature data. The algorithm is an iterative optimization that minimizes the objective function defined by Eq.(1). Then u_{ki} and C_i must satisfy the following equalities:

$$u_{ki} = \frac{1}{\sum_{L=1}^{C} \left[\frac{\|x_{i} - c_{L}\|^{2} + \alpha e^{-\eta (x_{i} - c_{L})^{2}}}{\|x_{i} - c_{k}\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}}} \right]^{\frac{1}{m-1}}}$$

(2)

1

$$c_{k} = \frac{\sum_{i=1}^{N} x_{i} u_{ki}^{m} (1 + e^{-\eta (x_{i} - \hat{c}_{k})^{2}})}{\left[\sum_{i=1}^{N} u_{ki}^{m} \left(1 + e^{-\eta (x_{i} - \hat{c}_{k})^{2}}\right)\right]}$$
(3)

Proof: We minimize the following equation using Lagrange method:

$$J_{m} = \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m} \|x_{i} - c_{k}\|^{2} + \alpha \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}}$$
(4)

where

 $u = (u_{ki})_{C \times N}$, $C = (c_1, c_2, ..., c_k)$, $\eta = 1 / 2\sigma^2$ and $\alpha = 1 / \sqrt{2\pi}$ is a constant.

For that, Eq.(4) can be rewritten as:

$$L_{m} = \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m} \|x_{i} - c_{k}\|^{2} +$$

$$\alpha \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}} + \sum_{i=1}^{N} \lambda_{i} (1 - \sum_{k=1}^{C} u_{ki}^{m})$$

$$\frac{\partial L_{m}}{\partial u_{ki}} = 0 \implies \sum_{k=1}^{C} \sum_{i=1}^{N} m u_{ki}^{m-1} \|x_{i} - c_{k}\|^{2} +$$

$$\alpha m \sum_{k=1}^{C} \sum_{i=1}^{N} u_{ki}^{m-1} e^{-\eta (x_{i} - c_{k})^{2}} + \sum_{i=1}^{N} \lambda_{i} (-1) = 0$$
(5)

$$\frac{\partial L_m}{\partial \lambda_i} = 0 \implies \sum_{k=1}^C u_{ki} - 1 = 0$$
(6)

$$u_{ki} = \left[\frac{\lambda_{i}}{m\left(\|x_{i} - c_{k}\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}}\right)}\right]^{\frac{1}{m-1}}$$
$$\left[\lambda_{i}\right]^{\frac{1}{m-1}} \left[1\right]^{\frac{1}{m-1}}$$

$$u_{ki} = \left[\frac{\lambda_{i}}{m}\right]^{\overline{m-1}} \left[\frac{1}{\|x_{i} - c_{k}\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}}}\right]^{m-1}$$

$$\sum_{k=1}^{c} u_{ki} = 1$$

$$\left[\frac{\lambda_{i}}{m}\right]^{\frac{1}{m-1}} \sum_{i=1}^{c} \left[\frac{1}{\|x_{i} - c_{k}\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}}}\right]^{\frac{1}{m-1}} = 1$$

$$u_{ki} = \sum_{L=1}^{C} \frac{\left(\left\| x_{i} - c_{L} \right\|^{2} + \alpha e^{-\eta (x_{i} - c_{L})^{2}} \right)^{\frac{1}{m-1}}}{\left(\left\| x_{i} - c_{k} \right\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}} \right)^{\frac{1}{m-1}}}$$

$$u_{ki} = \frac{1}{\sum_{L=1}^{C} \left[\frac{\|x_{i} - c_{L}\|^{2} + \alpha e^{-\eta (x_{i} - c_{L})^{2}}}{\|x_{i} - c_{k}\|^{2} + \alpha e^{-\eta (x_{i} - c_{k})^{2}}} \right]^{\frac{1}{m-1}}}{\frac{\partial L_{m}}{\partial c_{k}}} = 0$$
(7)

$$-2\sum_{i=1}^{m} u_{ki}^{m} (x_{i} - c_{k}) - 2\alpha (x_{i} - c_{k})\sum_{i=1}^{N} u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}} = 0$$

$$\sum_{i=1}^{N} u_{ki}^{m} x_{i} - \sum_{i=1}^{N} u_{ko}^{m} c_{k} + \sum_{i=1}^{N} x_{i} u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}} - \sum_{i=1}^{N} c_{k} u_{ki}^{m} e^{-\eta (x_{i} - c_{k})^{2}} = 0$$

$$\int c_k \left[\sum_{i=1}^N u_{ki}^m \left(1 + e^{-\eta (x_i - c_k)^2} \right) = \sum_{i=1}^N x_i u_{ki}^m \left(1 + e^{-\eta (x_i - c_k)^2} \right) \right]$$

We assume that C_k of the iterative process is \hat{C}_k and then $\hat{C}_k = C_{k-1}$ for the further iteration. Then we can rewrite the \hat{C}_k as follows:

$$c_{k} = \frac{\sum_{i=1}^{N} x_{i} u_{ki}^{m} (1 + e^{-\eta (x_{i} - \hat{c}_{k})^{2}})}{\left[\sum_{i=1}^{N} u_{ki}^{m} (1 + e^{-\eta (x_{i} - \hat{c}_{k})^{2}})\right]^{(8)}}$$

The process of finding the best clusters are continue to update the centres C_k and the membership u_{ki} using Eqs. (2) and (3) respectively. The R_k neighbors of the centres \hat{c}_k can be obtained using histogram preprocessing [29]. Algorithm:

- Initialize the membership matrix u_{ki}^t with random values between 0 and 1 such that the constraints in Equation
- (1) are satisfied.
- Input: initial centres \hat{c}_k , i=1,..., C, the data

$$p_i, i = 1..N$$

Repeat:

- Compute: u_{ki} and C_k using Eqs.(2) and (3).
- Until: $||u_{ki} u_{ki}^{t}|| \le \varepsilon$, where ε a certain tolerance value
- End Repeat

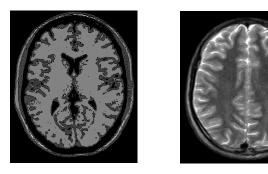
3 Experimental and Comparative Results

The experiments were performed on two different sets: one corrupted by (0%, 3%, 5%, 7%, 9%) salt and pepper noise and the image size is 129×129 pixels which are shown in Fig. 1(a), and Fig. 1(b), respectively [30]. The advantages of using digital phantoms rather than real image data for soft segmentation methods include prior knowledge of the true tissue types and control over image parameters such as modality, slice thickness, noise, and intensity in homogeneities. The quality of the segmentation algorithm is of vital importance to the segmentation process. The

comparison score AOM for each algorithm as proposed in [4] is defined as follows:

$$AOM = \left| \frac{A \cap A_{ref}}{A \cup A_{ref}} \right|$$

Where A represents the set of pixels belonging to a class as found by a particular method and A_{ref} represents the reference cluster pixels.



(a) (b) Fig.(1): Test images: (a) 3D simulated data, and (b) two original slices from the 3D simulated data (slice89 and slice 65).

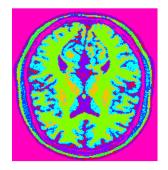


Fig. (2): Results of segmentation (noise 0%).

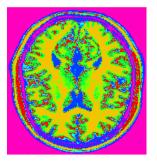


Fig. (3): Results of segmentation (noise 3%).

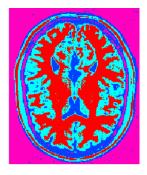


Fig. (4): Results of segmentation(noise 5%).

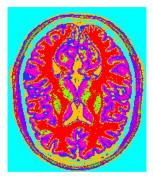


Fig. (5): Results of segmentation (noise 7%).

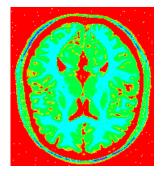


Fig. (6): Results of segmentation (noise 9%).

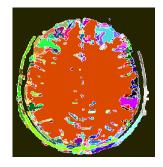


Fig. (7): Results of segmentation (noise 0%).

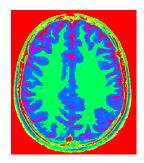


Fig. (8): Results of segmentation (noise 3%).

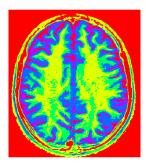


Fig. (9): Results of segmentation (noise 5%).

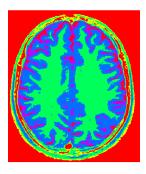


Fig. (10): Results of segmentation (noise 7%).

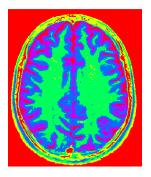


Fig. (11): Results of segmentation (noise 9%).

3.1 Experiment in the real image

We used a high-resolution T1-weighted MR phantom with slice thickness of 1mm obtained from the classical

simulated brain database of McGill University [36]. Table 1 shows AOM of WM with the proposed method is applied to MRI image various noise levels (0%, 3%, 5%, 7%, 9%) and RF levels 20% using, these results show that the proposed algorithms are very robust to noise and intensity homogeneities and inhomogeneities. According to Zijdenbos [19] statement that AOM> 0.7 indicates excellent agreement; the proposed method has desired performance in cortical segmentation. The best AOM is achieved for low noise and RF levels, for which values of AOM higher than 0.96.

Table 1: AOM for segmentations of WM on simulated T1weighted MRIs data in different noise and RF levels.

Noise/RF	20%
0%	0.98
3%	0.96
5%	0.96
7%	0.97
9%	0.96
7%	0.97
9%	0.96

3.2 Experiment on the Simulated MR data

Table 2 shows the corresponding accuracy scores (%) of the proposed and four other methods: standard FCM [1], Ahmed et al. [5], Chen and Zhang [6], and Kang et al. [23]for the nine classes. Obviously, the FCM gives the worst segmentation accuracy for all classes, whilethe proposed method gives the best. On the other hand, the method of Ahmed et al. [5], Chen and Zhang [6], and Kang et al. [21]acquire the good segmentation performance in case of classes 9, 4, and 1 respectively. Overall, the proposed method is more stable and achieves much better performance than the others in all different classes even with misleading of true tissue of validity indexes.

4 Conclusions

In this paper, we have proposed fuzzy c-means method that is based on gaussian to control the coincident clusters. The proposed algorithm incorporates the local spatial context into the standard FCM cluster algorithm and its complexity is reduced using initial centersas the prior information. It is formulated by modifying the objective function of the standard FCM algorithm to allow the labeling of a pixel to be influenced by other pixels and to suppress the noise effect during segmentation. We have tested the proposed algorithm on MRI images with 3%, 5%, 7% and 9% noise. We noted that the proposed method has desired performance in cortical segmentation. The superiority of the proposed algorithm is also demonstrated by comparing its performance with the standard FCM, Ahmed et al. [5], Chen and Zhang [6], and Kang et al. [21]. In addition, quantitative results are also given in our experiments. We noted that the segmentation accuracy of the proposed method is increased over the existing methods between21% and 14% for volumetric MR data (nine slices) over the best one. From the quantitative evaluation and the visual inspection, we can conclude that our proposed algorithm yields a robust and precise segmentation.

Table 2. Segmentation accuracy (%)(AOM) of the	
proposed and the existing methods on brain classes	•

proposed and the existing methods on brain elasses.						
Method	Class 1	Class 2	Class 3	Class 4	Class 5	
Standard KFCM	61.87	67	69.087	64.67	75.32	
Ahmed et al. [5]	77.55	61.14	78.83	73.88	67.96	
Chen and Zhang [21]	69.54	78.55	68.34	8201	78.65	
Kang et al. [6]	66.87	60.43	66.98	78.54	77.09	
The proposed method	83.76	78.45	80.09	90.34	83.56	

Table 2 (continued)

Method	Class 6	Class 7	Class 8	Class 9	Overall
Standard KFCM	47.96	73.99	13.12	90.66	62.63
Ahmed et al. [5]	61.87	89.21	15.27	81.97	67.52
Chen and Zhang [21]	81.98	80.7	18.54	78.54	69.355
Kang et al. [6]	80.98	66.87	16.43	79.09	65.92
The proposed method	68.12	89.64	59.34	96.98	81.12

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